

Equation	Solution
$\frac{dy}{dx} = \frac{f(x)}{g(y)}$	$\int g \, dy = \int f \, dx + C_1$ (C.1-1)
$\frac{dy}{dx} + f(x)y = g(x)$	$y = e^{-\int f \, dx} (\int e^{\int f \, dx} g \, dx + C_1)$ (C.1-2)
$\frac{d^2y}{dx^2} + a^2y = 0$	$y = C_1 \cos ax + C_2 \sin ax$ (C.1-3)
$\frac{d^2y}{dx^2} - a^2y = 0$	$y = C_1 \cosh ax + C_2 \sinh ax$ or (C.1-4a)
	$y = C_3 e^{+ax} + C_4 e^{-ax}$ (C.1-4b)
$\frac{1}{x^2} \frac{d}{dx} \left(x^2 \frac{dy}{dx} \right) + a^2y = 0$	$y = \frac{C_1}{x} \cos ax + \frac{C_2}{x} \sin ax$ (C.1-5)
$\frac{1}{x^2} \frac{d}{dx} \left(x^2 \frac{dy}{dx} \right) - a^2y = 0$	$y = \frac{C_1}{x} \cosh ax + \frac{C_2}{x} \sinh ax$ or (C.1-6a)
	$y = \frac{C_3}{x} e^{+ax} + \frac{C_4}{x} e^{-ax}$ (C.1-6b)
$\frac{d^2y}{dx^2} + a \frac{dy}{dx} + by = 0$	Solve the equation $n^2 + an + b = 0$, and get the roots $n = n_+$ and $n = n_-$. Then (a) if n_+ and n_- are real and unequal, (C.1-7a)
	$y = C_1 \exp(n_+x) + C_2 \exp(n_-x)$ (C.1-7a)
	(b) if n_+ and n_- are real and equal to n , (C.1-7b)
	$y = e^{nx}(C_1x + C_2)$ (C.1-7b)
	(c) if n_+ and n_- are complex: $n_{\pm} = p \pm iq$, (C.1-7c)
	$y = e^{px}(C_1 \cos qx + C_2 \sin qx)$ (C.1-7c)
$\frac{d^2y}{dx^2} + 2x \frac{dy}{dx} = 0$	$y = C_1 \int_0^x \exp(-\bar{x}^2) \, d\bar{x} + C_2$ (C.1-8)
$\frac{d^2y}{dx^2} + 3x^2 \frac{dy}{dx} = 0$	$y = C_1 \int_0^x \exp(-\bar{x}^3) \, d\bar{x} + C_2$ (C.1-9)
$\frac{d^2y}{dx^2} = f(x)$	$y = \int_0^x \int_0^{\bar{x}} f(\bar{x}) \, d\bar{x} \, d\bar{x} + C_1x + C_2$ (C.1-10)
$\frac{1}{x} \frac{d}{dx} \left(x \frac{dy}{dx} \right) = f(x)$	$y = \int_0^x \frac{1}{\bar{x}} \int_0^{\bar{x}} \bar{x} f(\bar{x}) \, d\bar{x} \, d\bar{x} + C_1 \ln x + C_2$ (C.1-11)
$\frac{1}{x^2} \frac{d}{dx} \left(x^2 \frac{dy}{dx} \right) = f(x)$	$y = \int_0^x \frac{1}{\bar{x}^2} \int_0^{\bar{x}} \bar{x}^2 f(\bar{x}) \, d\bar{x} \, d\bar{x} - \frac{C_1}{x} + C_2$ (C.1-12)
$\frac{d^2y}{dx^2} = h(y)$	$x = \int_0^y \frac{d\bar{y}}{\sqrt{2 \int_0^{\bar{y}} h(\bar{y}) \, d\bar{y}}} + C_2$ (C.1-13)
$x^3 \frac{d^3y}{dx^3} + ax^2 \frac{d^2y}{dx^2} + bx \frac{dy}{dx} + cy = 0$	(C.1-14)
	$y = C_1 x^{n_1} + C_2 x^{n_2} + C_3 x^{n_3}$, where the n_k are the roots of the equation $n(n-1)(n-2) + an(n-1) + bn + c = 0$, provided that all roots are distinct.

$$\cosh x = \frac{1}{2}(e^x + e^{-x}); \quad \sinh x = \frac{1}{2}(e^x - e^{-x}) \quad \cos x = \frac{1}{2}(e^{ix} + e^{-ix}); \quad \sin x = \frac{1}{2i}(e^{ix} - e^{-ix})$$